Solving linear and quadratic simultaneous equations

> **A LEVEL LINKS**
> **Scheme of work:** 1c. Equations – quadratic/linear simultaneous

Key points

- Make one of the unknowns the subject of the linear equation (rearranging where necessary).
- Use the linear equation to substitute into the quadratic equation.
- There are usually two pairs of solutions.

Examples

Example 1   Solve the simultaneous equations $y = x + 1$ and $x^2 + y^2 = 13$

$x^2 + (x + 1)^2 = 13$	**1** Substitute $x + 1$ for y into the second equation.
$x^2 + x^2 + x + x + 1 = 13$ $2x^2 + 2x + 1 = 13$	**2** Expand the brackets and simplify.
$2x^2 + 2x - 12 = 0$ $(2x - 4)(x + 3) = 0$ So $x = 2$ or $x = -3$	**3** Factorise the quadratic equation. **4** Work out the values of x.
Using $y = x + 1$ When $x = 2$, $y = 2 + 1 = 3$ When $x = -3$, $y = -3 + 1 = -2$	**5** To find the value of y, substitute both values of x into one of the original equations.
So the solutions are $x = 2$, $y = 3$ and $x = -3$, $y = -2$	

Check:
  equation 1: $3 = 2 + 1$ YES
      and $-2 = -3 + 1$ YES
 	
  equation 2: $2^2 + 3^2 = 13$ YES
     and $(-3)^2 + (-2)^2 = 13$ YES
	6 Substitute both pairs of values of x and y into both equations to check your answers.

Example 2   Solve $2x + 3y = 5$ and $2y^2 + xy = 12$ simultaneously.

$x = \dfrac{5 - 3y}{2}$	**1** Rearrange the first equation.
$2y^2 + \left(\dfrac{5-3y}{2}\right)y = 12$	**2** Substitute $\dfrac{5-3y}{2}$ for x into the second equation. Notice how it is easier to substitute for x than for y.
$2y^2 + \dfrac{5y - 3y^2}{2} = 12$	**3** Expand the brackets and simplify.
$4y^2 + 5y - 3y^2 = 24$	
$y^2 + 5y - 24 = 0$	
$(y + 8)(y - 3) = 0$	**4** Factorise the quadratic equation.
So $y = -8$ or $y = 3$	**5** Work out the values of y.
Using $2x + 3y = 5$ When $y = -8$, $2x + 3 \times (-8) = 5$, $x = 14.5$ When $y = 3$, $2x + 3 \times 3 = 5$, $x = -2$	**6** To find the value of x, substitute both values of y into one of the original equations.
So the solutions are $x = 14.5$, $y = -8$ and $x = -2$, $y = 3$	

Check:
 equation 1: $2 \times 14.5 + 3 \times (-8) = 5$ YES
        and $2 \times (-2) + 3 \times 3 = 5$ YES
 equation 2: $2 \times (-8)^2 + 14.5 \times (-8) = 12$ YES
        and $2 \times (3)^2 + (-2) \times 3 = 12$ YES
	7 Substitute both pairs of values of x and y into both equations to check your answers.

Practice

Solve these simultaneous equations.

1 $y = 2x + 1$
     $x^2 + y^2 = 10$

2 $y = 6 - x$
     $x^2 + y^2 = 20$

3 $y = x - 3$
     $x^2 + y^2 = 5$

4 $y = 9 - 2x$
     $x^2 + y^2 = 17$

5 $y = 3x - 5$
     $y = x^2 - 2x + 1$

6 $y = x - 5$
     $y = x^2 - 5x - 12$

7 $y = x + 5$
     $x^2 + y^2 = 25$

8 $y = 2x - 1$
     $x^2 + xy = 24$

9 $y = 2x$
     $y^2 - xy = 8$

10 $2x + y = 11$
     $xy = 15$

Extend

11 $x - y = 1$
     $x^2 + y^2 = 3$

12 $y - x = 2$
     $x^2 + xy = 3$

Answers

1 $x = 1, y = 3$

$x = -\dfrac{9}{5}, y = -\dfrac{13}{5}$

2 $x = 2, y = 4$
$x = 4, y = 2$

3 $x = 1, y = -2$
$x = 2, y = -1$

4 $x = 4, y = 1$

$x = \dfrac{16}{5}, y = \dfrac{13}{5}$

5 $x = 3, y = 4$
$x = 2, y = 1$

6 $x = 7, y = 2$
$x = -1, y = -6$

7 $x = 0, y = 5$
$x = -5, y = 0$

8 $x = -\dfrac{8}{3}, y = -\dfrac{19}{3}$

$x = 3, y = 5$

9 $x = -2, y = -4$
$x = 2, y = 4$

10 $x = \dfrac{5}{2}, y = 6$

$x = 3, y = 5$

11 $x = \dfrac{1 + \sqrt{5}}{2}, y = \dfrac{-1 + \sqrt{5}}{2}$

$x = \dfrac{1 - \sqrt{5}}{2}, y = \dfrac{-1 - \sqrt{5}}{2}$

12 $x = \dfrac{-1 + \sqrt{7}}{2}, y = \dfrac{3 + \sqrt{7}}{2}$

$x = \dfrac{-1 - \sqrt{7}}{2}, y = \dfrac{3 - \sqrt{7}}{2}$